Welcome to AP Calculus AB!

Attached is your Summer Packet that includes the prerequisite skills needed for success in Calculus. The first section of the packet contains practice problems, and the second section contains helpful notes and formulas for you to use while completing your packet. This assignment is **due the first day of school, September 6, 2022**. If you have any questions about this packet, please see me in room C8. I look forward to working with you in the fall!

Sincerely,

Dr. Coccimiglio

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Calculus - SUMMER PACKET

NAME:______________________

Summer + Math = (Best Summer Ever)^2

NO CALCULATOR!!!

Given $f(x) = x^2 - 2x + 5$, find the following.

1. $f(-2) =$

2. $f(x+2) =$

3. $f(x+h) =$

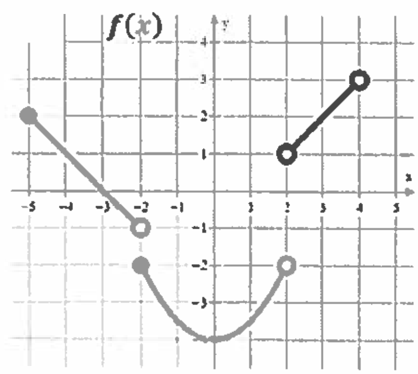
Use the graph $f(x)$ to answer the following.

4. $f(0) =$ $\quad$ $f(4) =$

$f(-1) =$ $\quad$ $f(-2) =$

$f(2) =$ $\quad$ $f(3) =$

$f(x) = 2$ when $x = ?$ $\quad$ $f(x) = -3$ when $x = ?$



Write the equation of the line meets the following conditions. Use point-slope form.
$y - y_1 = m(x - x_1)$

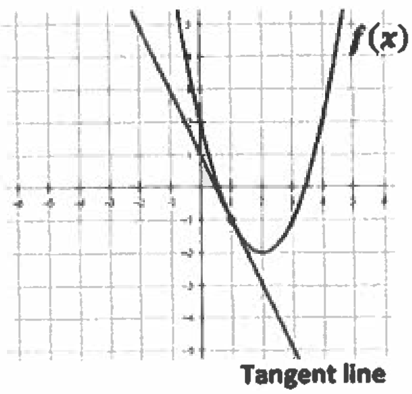
5. slope = 3 and $(4, -2)$

6. $m = -\frac{3}{2}$ and $f(-5) = 7$

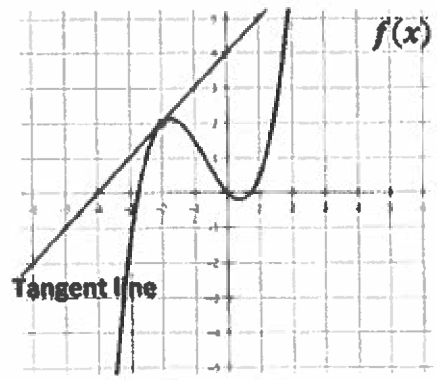
7. $f(4) = -8$ and $f(-3) = 12$

Write the equation of the tangent line in point slope form. $y - y_1 = m(x - x_1)$

8. The line tangent to $f(x)$ at $x = 1$



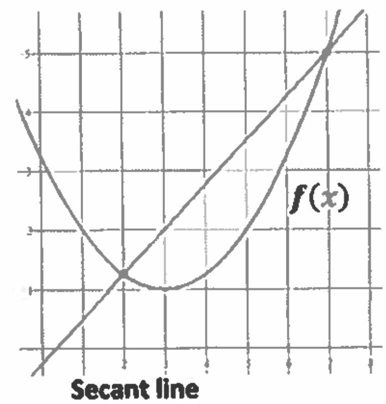
9. The line tangent to $f(x)$ at $x = -2$



MULTIPLE CHOICE! Remember slope $= \frac{y_2 - y_1}{x_2 - x_1}$

10. Which choice represents the slope of the secant line shown?

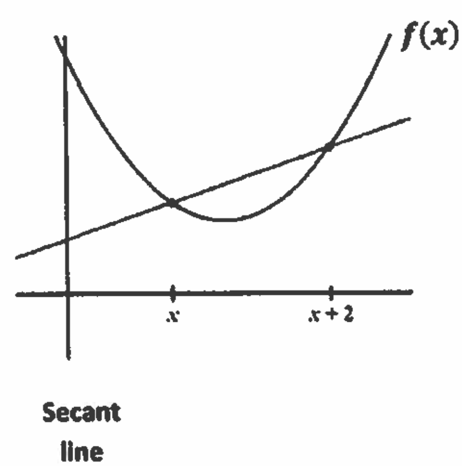
A) $\frac{7-2}{f(7)-f(2)}$ B) $\frac{f(7)-2}{7-f(2)}$ C) $\frac{7-f(2)}{f(7)-2}$ D) $\frac{f(7)-f(2)}{7-2}$



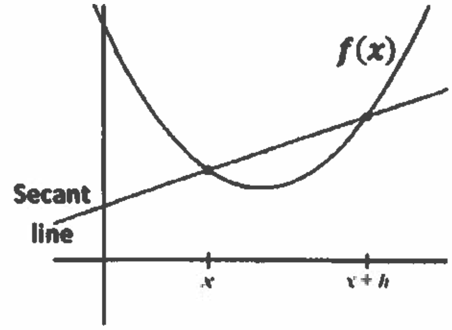
11. Which choice represents the slope of the secant line shown?

A) $\frac{f(x)-f(x+2)}{x+2-x}$ B) $\frac{f(x+2)-f(x)}{x+2-x}$ C) $\frac{f(x+2)-f(x)}{x-(x+2)}$

D) $\frac{x+2-x}{f(x)-f(x+2)}$



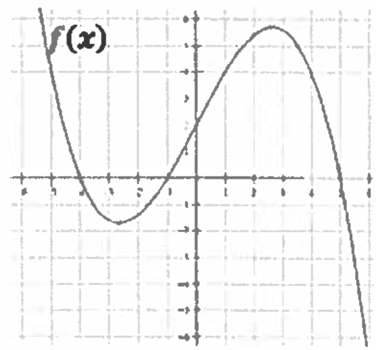
12. Which choice represents the slope of the secant line shown?



A) $\frac{f(x+h)-f(x)}{x-(x+h)}$ B) $\frac{x-(x+h)}{f(x+h)-f(x)}$ C) $\frac{f(x+h)-f(x)}{x+h-x}$

D) $\frac{f(x)-f(x+h)}{x+h-x}$

13. Which of the following statements about the function $f(x)$ is true?

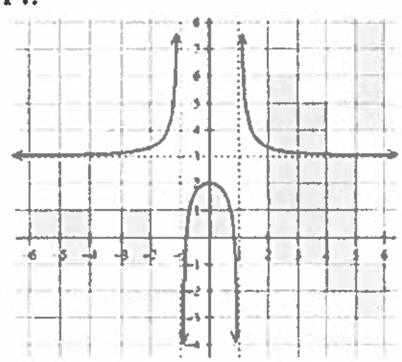


I. $f(2) = 0$
II. $(x + 4)$ is a factor of $f(x)$
III. $f(5) = f(-1)$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

Find the domain and range (express in interval notation). Find all horizontal and vertical asymptotes.

14.



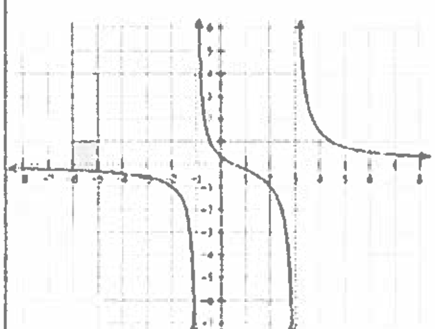
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

15.



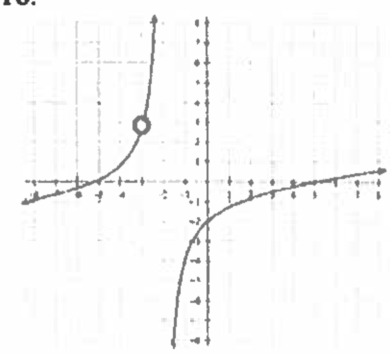
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

16.



Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

MULTIPLE CHOICE!

17. Which of the following functions has a vertical asymptote at $x = 4$?

(A) $\frac{x+5}{x^2-4}$

(B) $\frac{x^2-16}{x-4}$

(C) $\frac{4x}{x+1}$

(D) $\frac{x+6}{x^2-7x+12}$

(E) None of the above

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18. Consider the function: $(x) = \frac{x^2-5x+6}{x^2-4}$. Which of the following statements is true?

I. $f(x)$ has a vertical asymptote of $x = 2$
II. $f(x)$ has a vertical asymptote of $x = -2$
III. $f(x)$ has a horizontal asymptote of $y = 1$

(A) I only

(B) II only

(C) I and III only

(D) II and III only

(E) I, II and III

Rewrite the following using rational exponents. Example: $\frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$

19. $\sqrt[5]{x^3} + \sqrt[5]{2x}$	20. $\sqrt{x+1}$	21. $\frac{1}{\sqrt{x+1}}$
22. $\frac{1}{\sqrt{x}} - \frac{2}{x}$	23. $\frac{1}{4x^3} + \frac{1}{2}\sqrt[4]{x^3}$	24. $\frac{1}{4\sqrt{x}} - 2\sqrt{x+1}$

Write each expression in radical form and positive exponents. Example: $x^{-\frac{2}{3}} + x^{-2} = \frac{1}{\sqrt[3]{x^2}} + \frac{1}{x^2}$

25. $x^{-\frac{1}{2}} - x^{\frac{3}{2}}$	26. $\frac{1}{2}x^{-\frac{1}{2}} + x^{-1}$	27. $3x^{-\frac{1}{2}}$
28. $(x+4)^{-\frac{1}{2}}$	29. $x^{-2} + x^{\frac{1}{2}}$	30. $2x^{-2} + \frac{3}{2}x^{-1}$

Need to know basic trig functions in RADIANS! We never use degrees. You can either use the Unit Circle or Special Triangles to find the following.

31. $\sin\frac{\pi}{6}$	32. $\cos\frac{\pi}{4}$	33. $\sin 2\pi$
34. $\tan\pi$	35. $\sec\frac{\pi}{2}$	36. $\cos\frac{\pi}{6}$
37. $\sin\frac{\pi}{3}$	38. $\sin\frac{3\pi}{2}$	39. $\tan\frac{\pi}{4}$
40. $\csc\frac{\pi}{2}$	41. $\sin\pi$	42. $\cos\frac{\pi}{3}$
43. Find x where $0 \le x \le 2\pi$, $\sin x = \frac{1}{2}$	44. Find x where $0 \le x \le 2\pi$, $\tan x = 0$	45. Find x where $0 \le x \le 2\pi$, $\cos x = -1$

Solve the following equations. Remember $e^0 = 1$ and $\ln 1 = 0$.

46. $e^x + 1 = 2$	47. $3e^x + 5 = 8$	48. $e^{2x} = 1$
49. $\ln x = 0$	50. $3 - \ln x = 3$	51. $\ln(3x) = 0$
52. $x^2 - 3x = 0$	53. $e^x + xe^x = 0$	54. $e^{2x} - e^x = 0$

Solve the following trig equations where $0 \le x \le 2\pi$.

55. $\sin x = \frac{1}{2}$	56. $\cos x = -1$	57. $\cos x = \frac{\sqrt{3}}{2}$
58. $2\sin x = -1$	59. $\cos x = \frac{\sqrt{2}}{2}$	60. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$
61. $\tan x = 0$	62. $\sin(2x) = 1$	63. $\sin\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2}$

For each function, determine its domain and range.

Function	Domain	Range
64. $y = \sqrt{x-4}$		
65. $y = (x-3)^2$		
66. $y = \ln x$		
67. $y = e^x$		
68. $y = \sqrt{4-x^2}$		

Simplify.

69. $\frac{\sqrt{x}}{x}$	70. $e^{\ln x}$	71. $e^{1+\ln x}$

72. $\ln 1$

73. $\ln e^7$

74. $\log_3 \frac{1}{3}$

75. $\log_{1/2} 8$

76. $\ln \frac{1}{2}$

77. $27^{\frac{2}{3}}$

78. $\left(5a^{2/3}\right)\left(4a^{3/2}\right)$

79. $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$

80. $\left(4a^{5/3}\right)^{3/2}$

If $f(x) = \{(3,5),(2,4),(1,7)\}$, $h(x) = \{(3,2),(4,3),(1,6)\}$, $g(x) = \sqrt{x-3}$, $k(x) = x^2 + 5$, **then determine each of the following.**

81. $(f+h)(1)$

82. $(k-g)(5)$

83. $f(h(3))$

84. $g(k(7))$

85. $h(3)$

86. $g(g(9))$

87. $f^{-1}(4)$

88. $k^{-1}(x)$

89. $k(g(x))$

90. $g(f(2))$

Things to Know for Calculus

TRIGONOMETRY

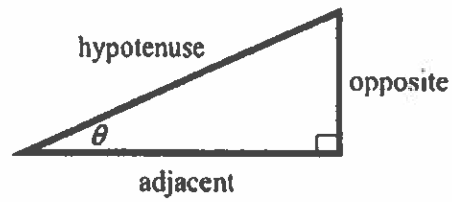
Trig Functions

$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$

Reciprocal Functions

$$\csc\theta = \frac{1}{\sin\theta} = \frac{hyp}{opp} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{hyp}{adj} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{adj}{opp}$$

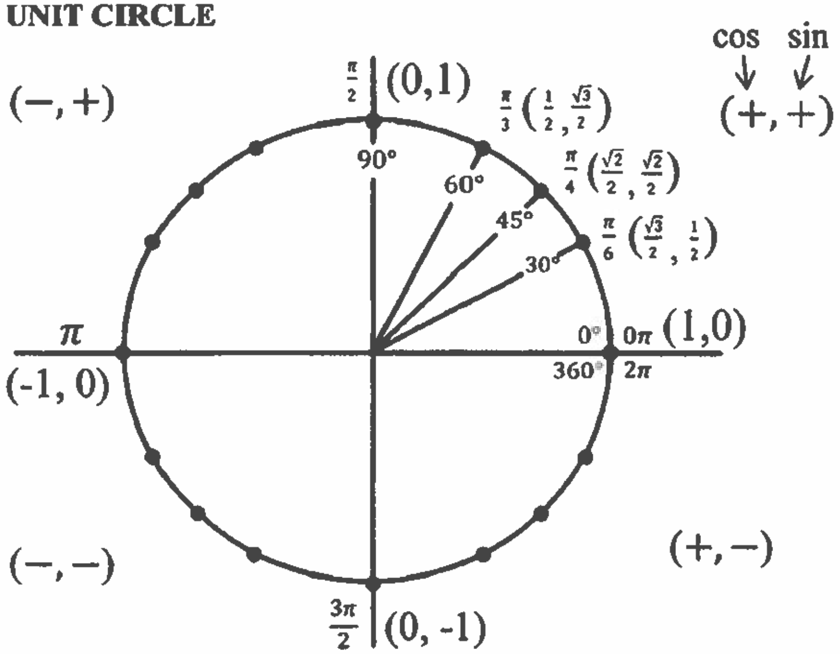
SOH-CAH-TOA



TEST ONLY USES RADIANS!

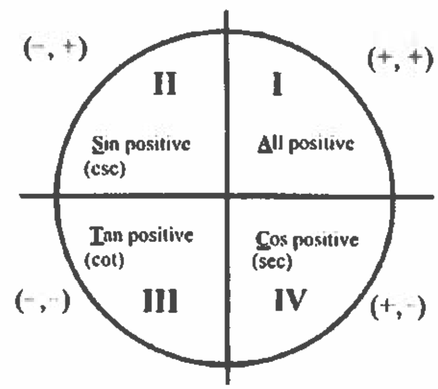
Must know trig values of special angles $0\pi, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ using Unit Circle or Special Right Triangles.

UNIT CIRCLE



To help remember the signs in each quadrant

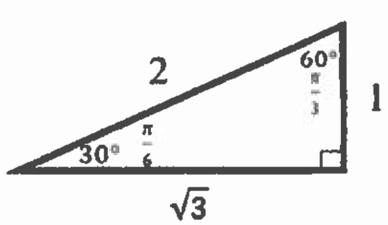
All **S**tudents **T**ake **C**alculus



SPECIAL RIGHT TRIANGLES

30° − 60° − 90° Triangles

Which are $\frac{\pi}{6} - \frac{\pi}{3} - \frac{\pi}{2}$ Triangles

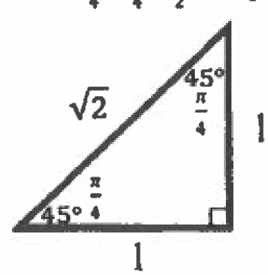


Find $\tan\left(\frac{\pi}{6}\right)$

$$\tan\left(\frac{\pi}{6}\right) = \frac{opp}{adj} = \frac{1}{\sqrt{3}} \text{ simplify to } \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

45° − 45° − 90° Triangles

Which are $\frac{\pi}{4} - \frac{\pi}{4} - \frac{\pi}{2}$ Triangles

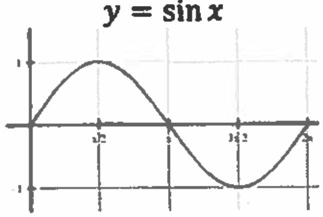


Find $\sin\left(\frac{\pi}{4}\right)$

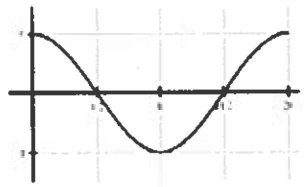
$$\sin\left(\frac{\pi}{4}\right) = \frac{opp}{hyp} = \frac{1}{\sqrt{2}} \text{ simplify to } \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Graphs of trig functions

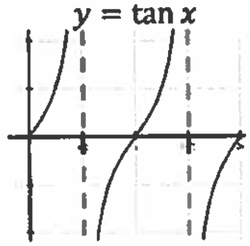
$y = \sin x$



$y = \cos x$



$y = \tan x$



Inverse Trig Function

$\sin^{-1}\theta$ is the same as $\arcsin\theta$

$\sin^{-1}\theta = \left(\frac{\sqrt{3}}{2}\right)$ means what angle has a sine value of $\frac{\sqrt{3}}{2}$

that means $\theta = \frac{\pi}{3} \pm 2\pi n$ *or* $\frac{2\pi}{3} \pm 2\pi n$

Since θ has infinite answers then it isn't a function. Bummer. To make it a function we define inverses like:

sin/csc and tan/cot use quadrant I and IV for inverses
cos/sec use quadrant I and II for inverses

So… $\theta = \frac{\pi}{3}$ because it is in the first quadrant

Trig Identities

There are a bunch, but you really only need to know Pythagorean Identity. $\sin^2 x + \cos^2 x = 1$

Subtract $\sin^2 x$ to get $\cos^2 x = 1 - \sin^2 x$ or subtract $\cos^2 x$ to get $\sin^2 x = 1 - \cos^2 x$

Divide by $\sin^2 x$ to get $1 + \cot^2 x = \csc^2 x$ or divide by $\cos^2 x$ to get $\tan^2 x + 1 = \sec^2 x$

GEOMETRY

FORMULAS

AREA

Triangle $= \frac{1}{2}bh$

Circle $= \pi r^2$

Trapezoid $= \frac{1}{2}(b_1 + b_2)h$

CIRCUMFERENCE

Circle $= 2\pi r$

SURFACE AREA

Sphere $= 4\pi r^2$

LATERAL AREA

Cylinder $= 2\pi rh$

VOLUME

Sphere $= \frac{4}{3}\pi r^3$

Cylinder $= \pi r^2 h$

Cone $= \frac{1}{3}\pi r^2 h$

Prism $= Bh$

Pyramid $= \frac{1}{3}Bh$

B is the area of the base

DISTANCE FORMULA

The distance between two points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

ALGEBRA

Linear Functions

Slope

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

y-intercept Form

(slope-intercept Form)

$$y = mx + b$$

Point Slope Form

$$y - y_1 = m(x - x_1)$$

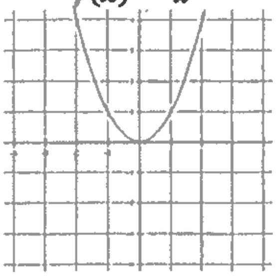
Parallel Lines
Have the same slope

Perpendicular Lines
Have the opposite reciprocal slopes

Functions

Quadratic Function

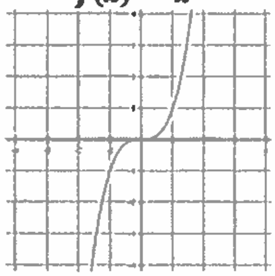
$f(x) = x^2$



$y = a(x-h)^2 + k$

Cubic Function

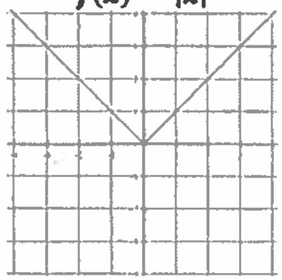
$f(x) = x^3$



$y = a(x-h)^3 + k$

Absolute Value

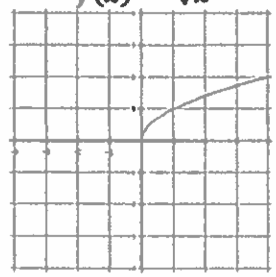
$f(x) = |x|$



$y = a|x-h| + k$

Square Root Function

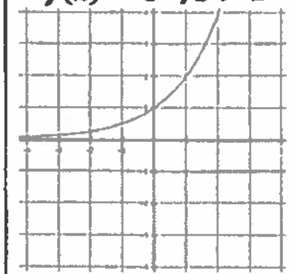
$f(x) = \sqrt{x}$



$y = a\sqrt{x-h} + k$

Exponential Function

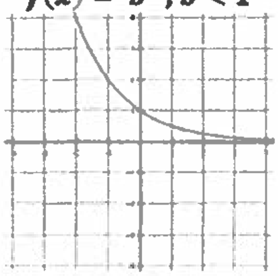
$f(x) = b^x, b > 1$



$y = a \cdot b^{(x-h)} + k$

Exponential Function

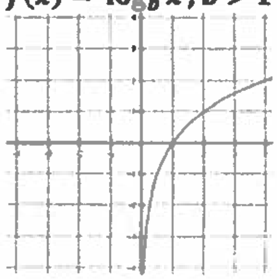
$f(x) = b^x, b < 1$



$y = a \cdot b^{(x-h)} + k$

Logarithmic Function

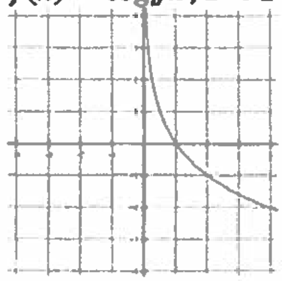
$f(x) = \log_b x, b > 1$



$y = a\log_b(x-h) + k$

Logarithmic Function

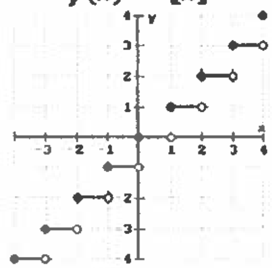
$f(x) = \log_b x, b < 1$



$y = a\log_b(x-h) + k$

Greatest Integer

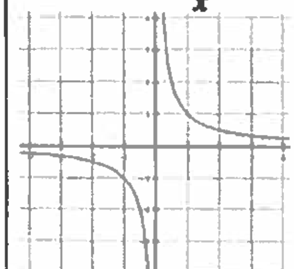
$f(x) = [\![x]\!]$



$y = a[\![x-h]\!] + k$

Rational Function

$f(x) = \frac{1}{x}$



$y = \frac{a}{x-h} + k$

Translations

All functions move the same way!

Given the parent function $y = x^2$

Move up 4	Move down 3	Move left 2	Move right 1	Move left 2 and down 3
$y = x^2 + 4$	$y = x^2 - 3$	$y = (x+2)^2$	$y = (x-1)^2$	$y = (x+2)^2 - 3$

To flip (reflect) the function vertically $y = -x^2$

To flip (reflect) the function horizontally $y = (-x)^2$

So $f(x) = -\sqrt{x-3} + 1$ is a square root function reflected vertically, shifted right 3 and up 1

Notation

Notice open parenthesis ( ) versus closed [ ]

Inequality		Interval
$-3 < x \le 5$	$\longleftrightarrow$	$(-3,5]$
$-3 \le x \le 5$	$\longleftrightarrow$	$[-3,5]$
$-3 < x < 5$	$\longleftrightarrow$	$(-3,5)$
$-3 \le x < 5$	$\longleftrightarrow$	$[-3,5)$

Infinity is always open parenthesis

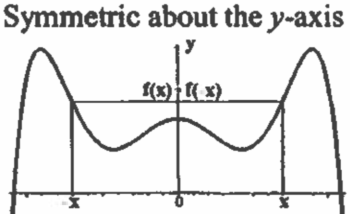
Inequality		Interval
$x < 3$	$\longleftrightarrow$	$(-\infty,3)$
$x \le 3$ or $x > 5$	$\longleftrightarrow$	$(-\infty,3](5,\infty)$
$x \ne 3$	$\longleftrightarrow$	$(-\infty,3)(3,\infty)$
all Real numbers	$\longleftrightarrow$	$(-\infty,\infty)$

Even and Odd Functions

EVEN

$f(-x) = f(x)$

Symmetric about the y-axis

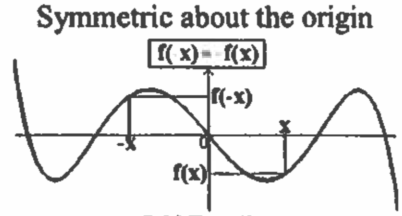


Even Function

ODD

$f(-x) = -f(x)$

Symmetric about the origin

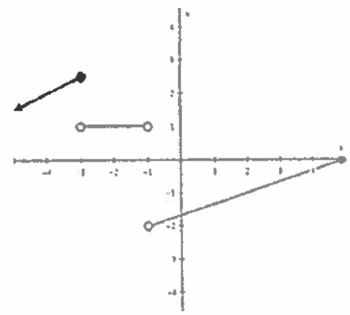


Odd Function

Domain and Range

Domain = all possible x values

Range = all possible y values

Algebraically You can't divide by zero You can't square root a negative		Graphically Just look at it	
$y=\sqrt{2x+5}$ D: $[-\frac{5}{2},\infty)$	$y=\dfrac{x^2-1}{x^2+7x+12}$ D: $(-\infty,-4)(-4,-3)(-3,\infty)$		D: $(-\infty,-1)(-1,5]$ R: $(-\infty,2.5]$

Finding zeros

Must be able to factor and use the quadratic formula: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

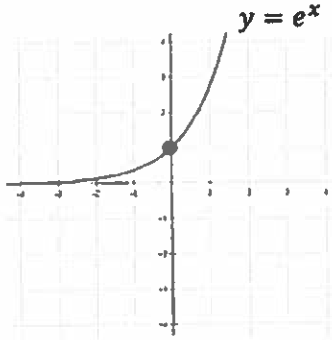
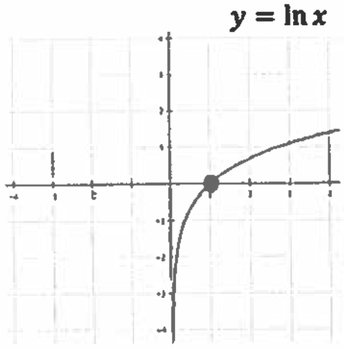
Special products

Sum of cubes: $a^3+b^3=(a+b)(a^2-ab+b^2)$

Difference of cubes: $a^3-b^3=(a-b)(a^2+ab+b^2)$

Exponential and Logarithmic Properties

The exponential function b^x of base b is one-to-one which means it has an inverse which is called the logaritmic function of base b or logarithm of base b which is denoted $\log_b x$ which reads "the logarithm of base b of x" or "log base b of x". So…

BASE e	
$y=e^x$ 	KNOW! $e^0=1$ Horizontal asymptote at $y=0$
$y=\ln x$ 	KNOW! $\ln 1=0$ Vertical asymptote at $x=0$

$$y=\log_b x \longleftrightarrow x=b^y$$

Exponential		Logarithmic
$b^x b^y=b^{x+y}$	Product Rule	$\log_b xy=\log_b x+\log_b y$
$\frac{b^x}{b^y}=b^{x-y}$	Quotient Rule	$\log_b\left(\frac{x}{y}\right)=\log_b x-\log_b y$
$(b^x)^y=b^{xy}$	Power Rule	$\log_b x^y=y\log_b x$
$b^{-x}=\frac{1}{b^x}$		$\log_b\left(\frac{1}{x}\right)=-\log_b x$
$b^0=1$		$\log_b 1=0$
$b^1=b$		$\log_b b=1$
	Change of Base	$\log_b x=\frac{\log_c x}{\log_c b}$
	Natural Log	$\log_e x=\ln e$
	Common Log	$\log_{10} x=\log x$